# The Neutron Spectrum

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February 10, 2016

# Review: SLBW Capture

$$\bar{\sigma}_x(E) = \sigma_0(E)\frac{\Gamma_{x,i}}{\Gamma_i}\psi(u,\alpha,\beta)$$

where

$$\psi(u,\alpha,\beta) = \frac{1}{\beta\sqrt{\pi}}\int_{-\infty}^{\infty} dv \frac{1}{1+v^2} \times \exp\left\{-\frac{(v-u)^2}{\beta^2}\left[1 - \frac{1}{2}\alpha(v-u) + \frac{5}{16}\alpha^2(v-u)^2 + \ldots\right]\right\}$$

$$\alpha = \frac{\Gamma_i}{2E} \quad \text{and} \quad \beta = \frac{2\Gamma_D}{\Gamma_i} = 4\sqrt{\frac{E_i kT}{A}}\frac{1}{\Gamma_i} \ .$$

# Review: SLBW Scattering

$$\bar{\sigma}_e(E) = 4\pi a^2 + \sigma_0(E)\frac{2a}{\lambda}\phi(u,\alpha,\beta) + \sigma_0(E)\frac{\Gamma_{n,i}}{\Gamma_i}\psi(u,\alpha,\beta) \quad (1)$$

where

$$\phi(u,\alpha,\beta) = \frac{1}{\beta\sqrt{\pi}}\int_{-\infty}^{\infty} dv \frac{v}{1+v^2} \times \quad (2)$$

$$\exp\left\{-\frac{(v-u)^2}{\beta^2}\left[1 - \frac{1}{2}\alpha(v-u) + \frac{5}{16}\alpha^2(v-u)^2 + \ldots\right]\right\} \quad (3)$$

**What is a neutron *spectrum*?**

The neutron population depends on energy, $n(E)$. Multiplying by speed produces $\phi(E) = v(E)n(E)$. This is called the spectrum.

## Classifying the neutron spectrum

1. Fission range ($E > 0.5$ MeV)
2. Slowing-down range ($1$ eV $< E < 50$ keV)
3. Thermal range ($E < 1$ eV)

# Introduction

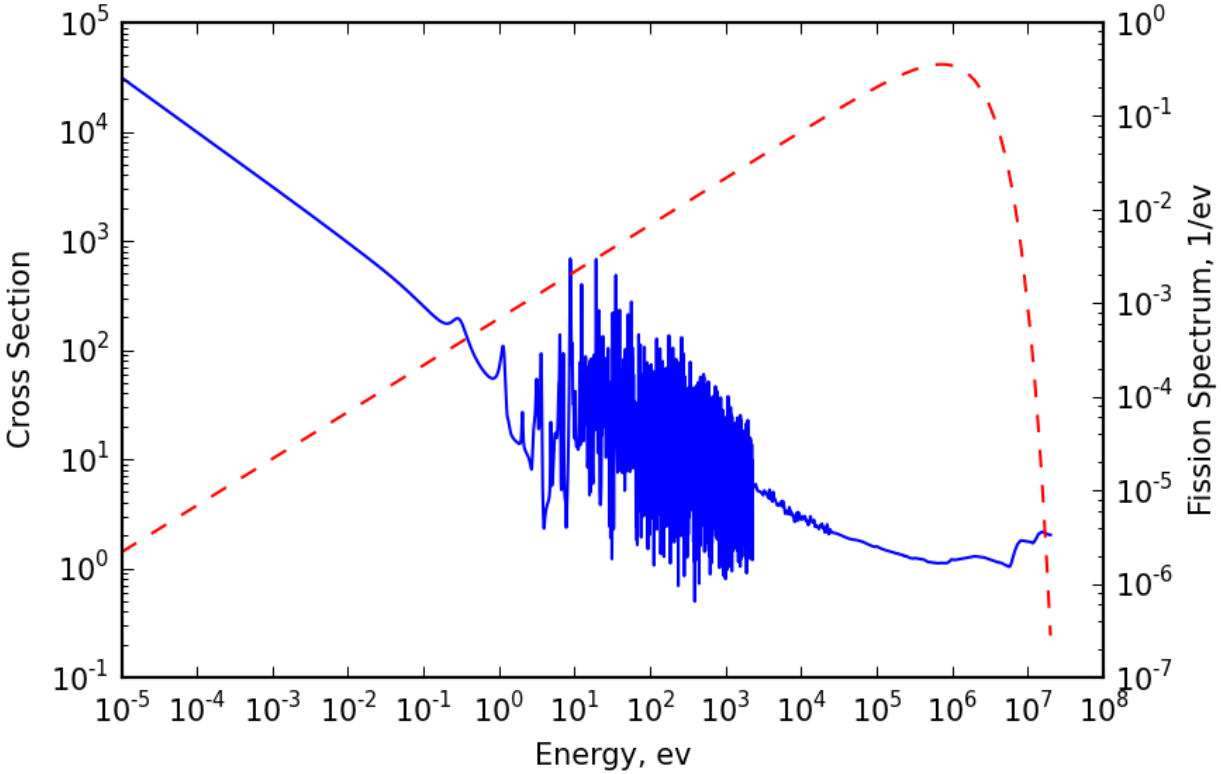


Figure: Fission cross section (blue) and fission spectrum (red) of uranium-235.

# Fission Energy Range

- Highest energy range
- $E > 0.5$ MeV
- Neutrons created from fission

$$\phi(E)dE \approx \frac{\chi(E)}{\Sigma_t(E)} \times \text{ constant}$$

# Slowing-Down Range

- Intermediate (epi-thermal) energy range
- 1 eV $< E <$ 50 keV
- Resonances live here
- Interactions dominated by elastic scattering and resonance absorption

$$\phi(E) = \frac{\left[\Sigma_a(E_1) + \Sigma_s^H\right] E_1 \phi(E_1)}{\left[\Sigma_a(E) + \Sigma_s^H\right] E} \times \exp\left[-\int_E^{E_1} \frac{\Sigma_a(E')}{\left[\Sigma_a(E') + \Sigma_s^H\right] E'} dE'\right].$$

# Thermal Range

- Low energy range
- $E < 1$ eV
- Neutrons in thermal equilibrium with medium
- Few if any resonances
- Characterized by Maxwell-Boltzmann dist. with effective temperature

$$\phi(E) = 2\sqrt{\frac{E}{\pi}}\left(\frac{1}{kT}\right)^{3/2}\exp\left(-\frac{E}{kT}\right).$$

# Characteristic Neutron Spectrum

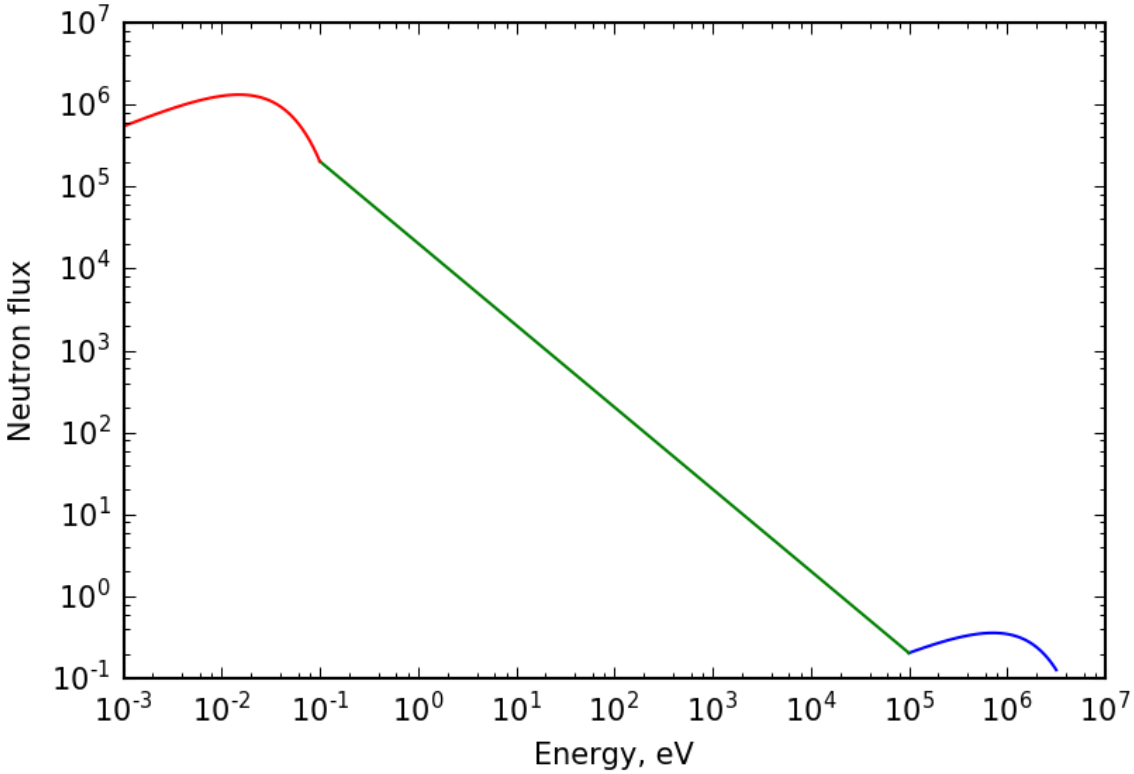


Figure: Rough caricature of a typical neutron spectrum.

# Slowing Down and Resonance Absorption

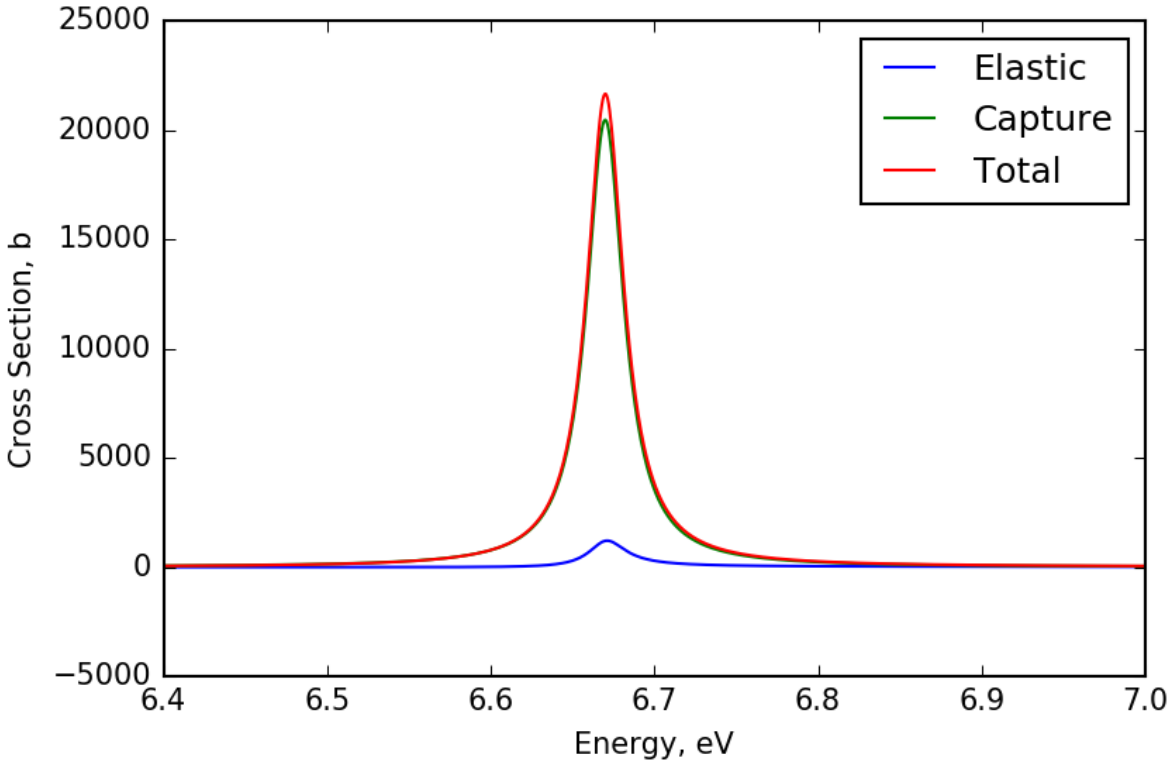


Figure: Resonance cross section at 0K.

# Slowing Down and Resonance Absorption

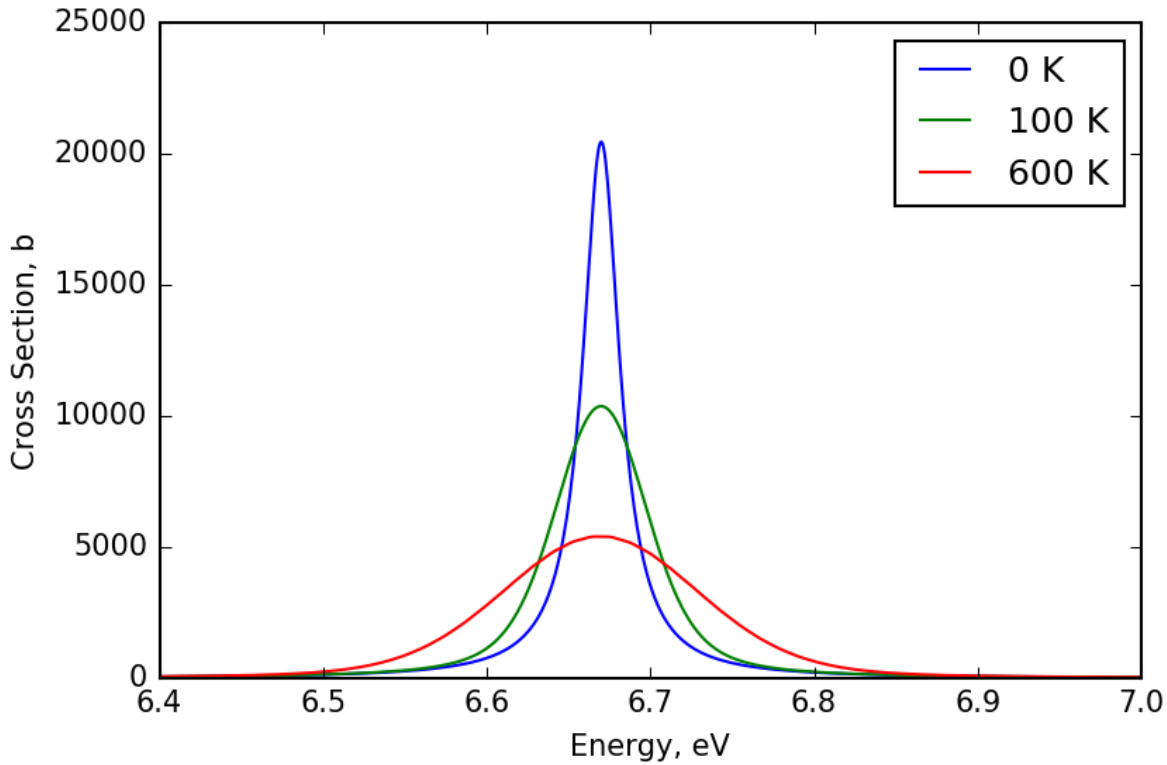


Figure: Doppler broadening of capture resonance.

# Slowing Down and Resonance Absorption

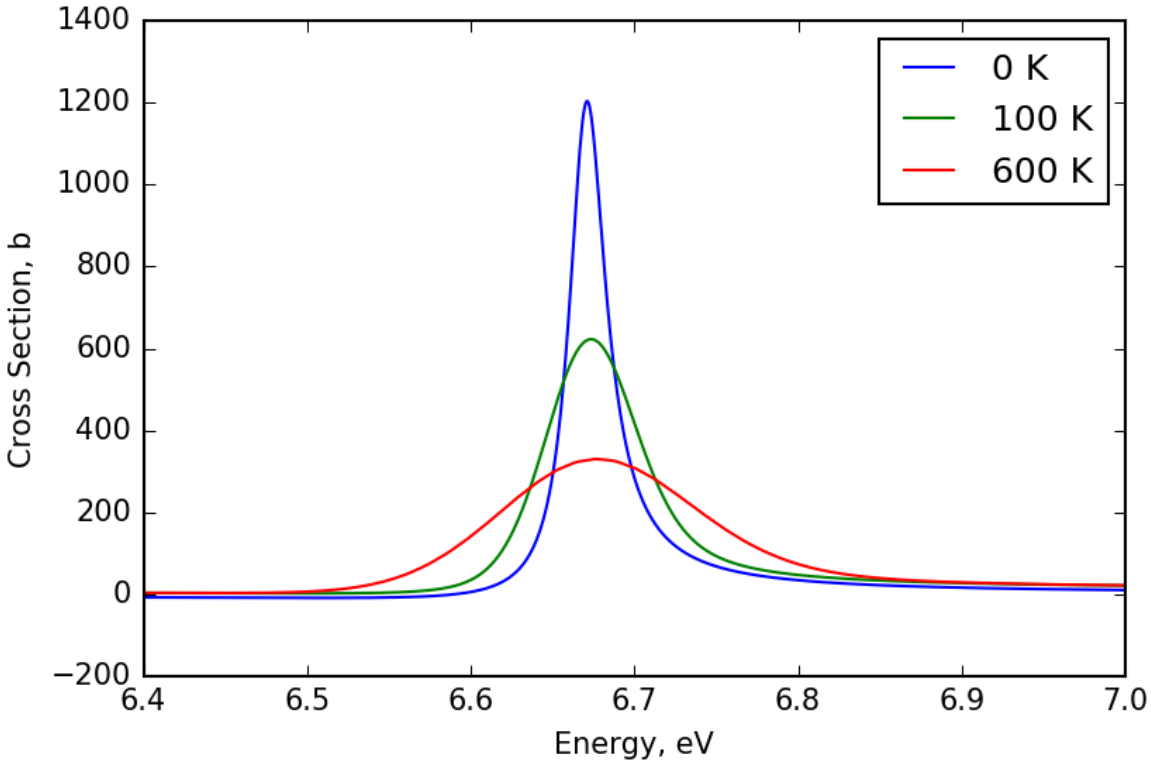


Figure: Doppler broadening of scatter resonance.

# Slowing Down and Resonance Absorption

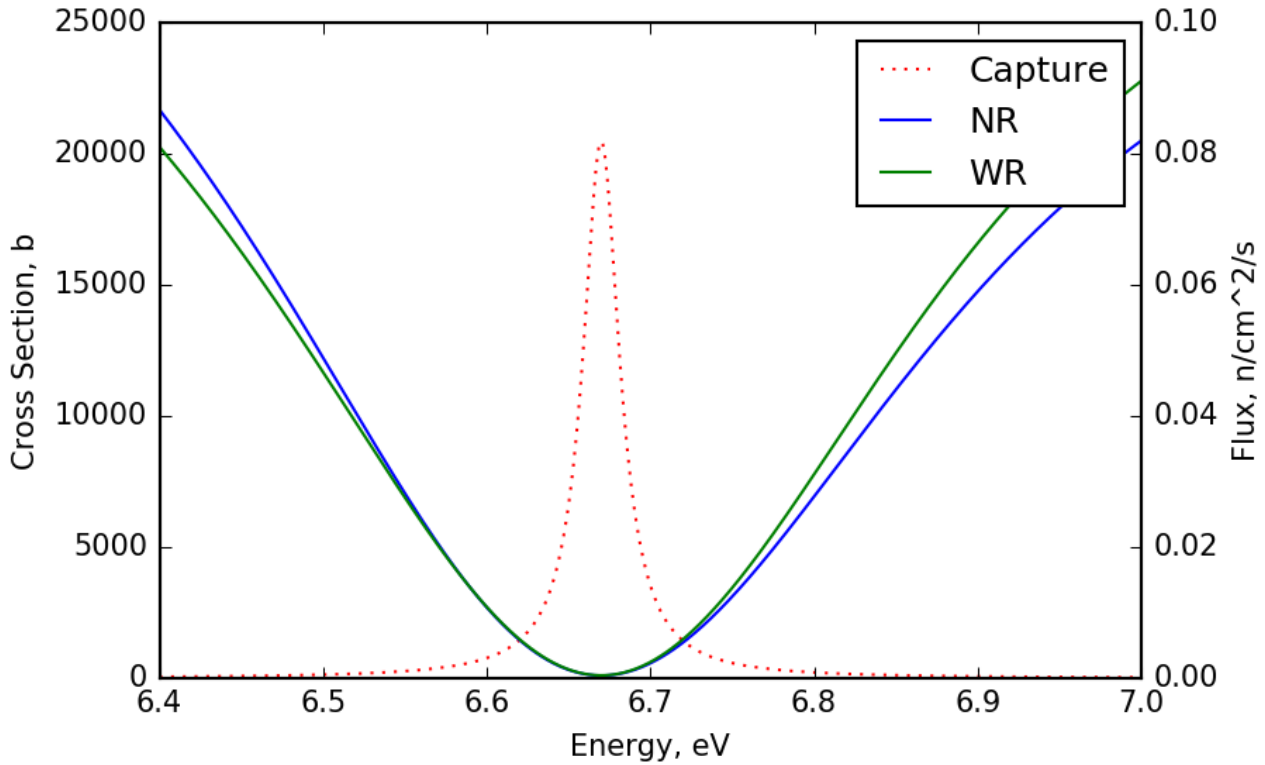


Figure: Self shielding of neutron flux in resonance.

# Slowing Down and Resonance Absorption

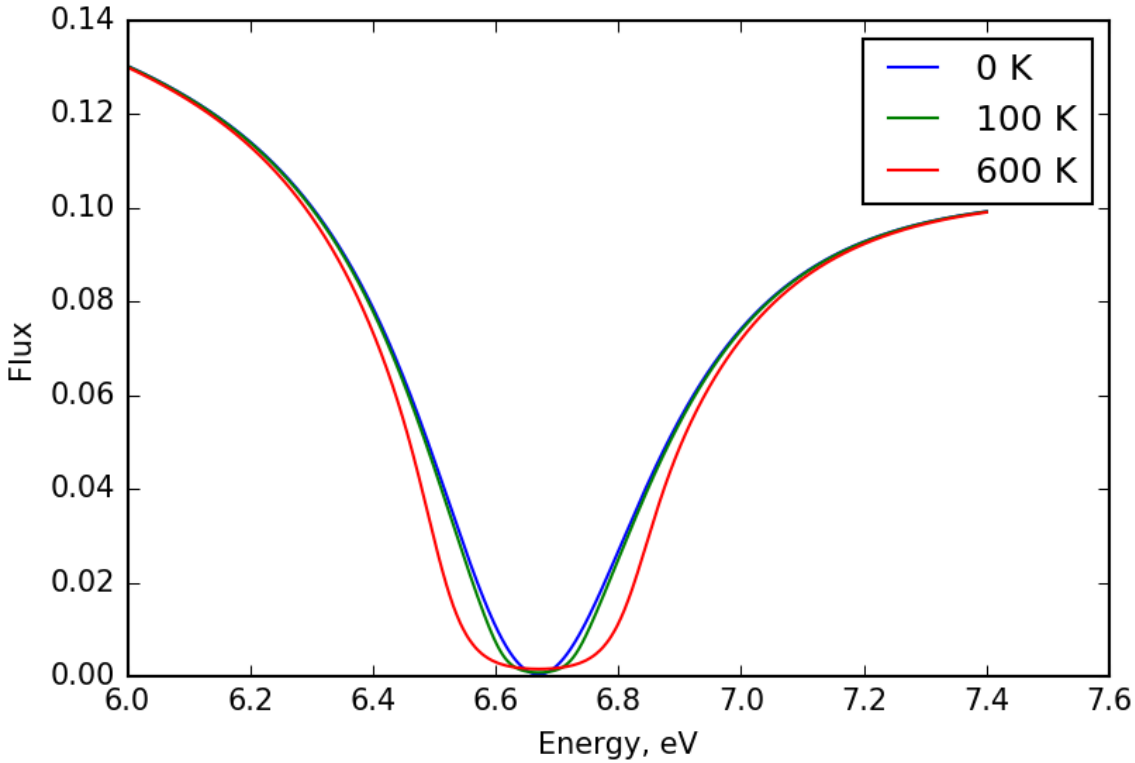


Figure: Self shielding of neutron flux in resonance.